Local Linearity Discovery

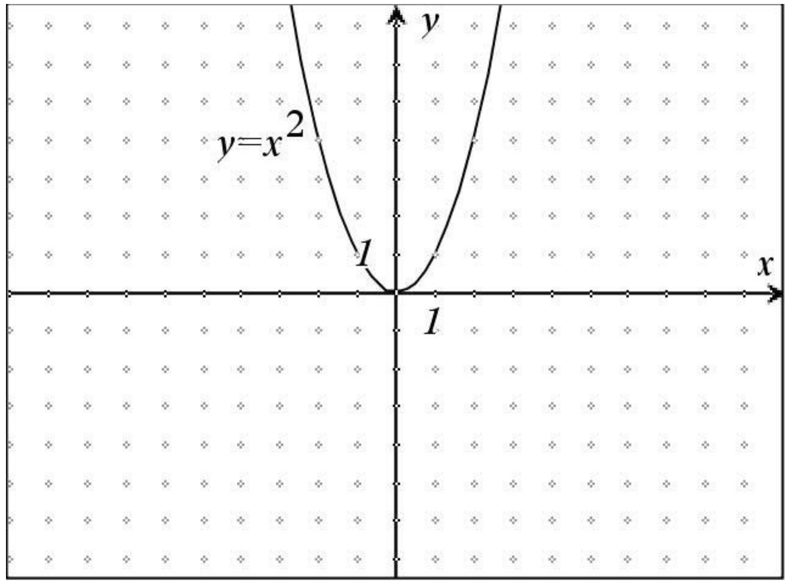
LocalLinearity.tns

Name ________________________

Class ________________________

Part 1 – Draw A Tangent Line by Hand

On the graph to the right, draw a line tangent to $y = x^2$ in the first quadrant.



1. Approximate the slope of the line. Show your work.

2. Write the equation of your line.

Part 2 – Draw and Explore Tangent Line with Technology

Press (⌂ on) and select **New Document**. (Unless you want to save what you are working on, arrow over to "No" and press (enter).)

Select **Add Graphs** by using arrows and pressing (enter) or [click].

Graph the equation $y = x^2$ in **f1**(x) by pressing [X] (x^2) (enter).

Draw a tangent line on the graph in the first quadrant. Select **MENU > Points & Lines > Tangent**. Glide your finger across the Touchpad to use it as a mouse or click the arrows on the side of it to move the cursor to your desired point and press (enter).

Zoom in and observe the behavior. Press **Menu > Window/Zoom > Zoom – In**. A magnifying glass with a plus sign will appear. Position this over the point of tangency and press (enter). You have zoomed in by a factor of 2. Press enter a few more times over that point.

3. Write your observations of how your tangent line and the graph **f1**(x) = x^2 compare when examined close up.

4. **Conjecture –** Will this type of behavior occur for all other functions? Explain your reasoning.

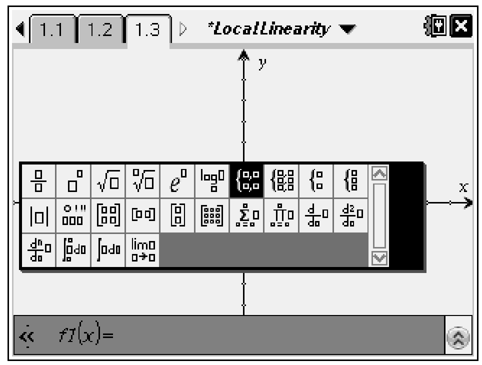
(You may want to try it for another function. You can choose your own function or try **f2**(x) = sin(x) on another *Graphs* page. Press (trig)[X](enter) to graph sin(x).
Press (ctrl)(doc▾), [+page], to insert another page.
TIP: Press (tab) to go back to the entry line. Arrow up to see previously entered functions.)

Part 3 – Graph a piecewise function to explore local linearity

A function is said to be linear over an interval (i.e. locally linear over a small interval) if the slope is constant. Let's discover if all functions have a constant slope when they are examined in a small enough interval.

To graph $y = \begin{cases} x^2, x < 2 \\ 2x, x \geq 2 \end{cases}$, first insert another *Graph* page.

The piecewise function template can be found by pressing [template key]. Arrow over to select the 2-piece piecewise function template and press enter. Then, press X x² tab X ctrl = ▸ enter 2 tab 2 X tab X ctrl = ▾ enter 2 enter.



Discover if all functions have the property of local linearity by zooming in on the point (2, 4). This point is called a *cusp*.

- Explain your observations. Use words like "slope" and "local linearity" to explain if, in the neighborhood of (2, 4), the function becomes one straight line. (Use **MENU > Window/Zoom > Window Settings** to ensure that your zoomed-in window still contains (2, 4). It may also help to show the end values of the axes if they are not already displayed; select **MENU > View > Show Axes End Values**.)

Part 4 – Graph another piecewise function

To explore if all piecewise functions lack the property of local linearity, on a new *Graphs* page zoom in on (2, 4) of the function $f(x) = \begin{cases} x^2 & , x < 2 \\ 4x - 4, & x \geq 2 \end{cases}$.

5. Does this function appear to be locally linear in the neighborhood of (2, 4)? Compare and contrast this function to the one graphed and explored in Problem 3.

Part 5 – Conclusion

You know the definition of slope is $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$. For the function $f(x)$, this can be written as $\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$. If you were finding the slope of function in the interval of a repeatedly zoomed in graph, describe what happens to $\Delta x = x_2 - x_1$.